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Macroeconomic Implications of Inequality and Income Risk*

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November 16, 2021

Abstract

We explore the long-run relationship between income risk, inequality, and the macroeconomy in an overlapping-generations model in which households face uncertain streams of labor income and returns on their savings. To manage those risks, households can apportion their savings to a bond, whose return is safe and identical across households, and a productive asset, whose return is uncertain and can differ persistently across households. We find that greater polarization in households' labor income and returns on their savings generally accentuates households' demand for risk-free assets and the compensation they require for bearing risk, leading to higher measured income and wealth inequality, a lower risk-free real interest rate, and higher risk premiums. These findings suggest that the factors behind the observed rise in inequality over the past few decades might have contributed to the observed fall in the risk-free real interest rate and widening gap between the risk-free real interest rate and the rate of return on capital. We also find that the magnitude of the decline in the risk-free real interest rate and offsetting rise in risk premiums depend importantly on the source of income polarization, with the effects being especially large when greater inequality is caused by increased dispersion in returns on risky assets. Thus, the macroeconomic implications not only depend on the amount of inequality, but also the source of this inequality.

JEL Codes: D31, D33, E21, E25, J11.

Keywords: income and wealth inequality, heterogeneous returns, risk-free real interest rate, risk premium.

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1 Introduction

Income and wealth have become increasingly concentrated in the hands of the richest households in the past few decades. Analysis based on the Federal Reserve's Survey of Consumer Finances (SCF) suggests that the top 10 percent of U.S. households now receives almost half of aggregate income and owns nearly 80 percent of aggregate net worth, both up notably from earlier decades.¹ In this paper, we explore how a rise in income polarization that exacerbates wealth inequality affects the macroeconomy, and whether the associated effects depend on the drivers of income polarization. In particular, we quantify how polarization in households' labor earnings and in the return on their savings affects real economic activity, saving decisions, and interest rates in a general-equilibrium, overlapping-generations model.

Our results suggest that increased polarization in labor income and the returns on savings are plausible contributors to the observed rise in inequality measures, the estimated decline in the trend of the risk-free real interest rate, and the apparent widening of the gap between the risk-free real interest rate and the real rate of return on capital over the past few decades (documented below). Intuitively, greater income heterogeneity accentuates households' desire to save against potential changes in income. The increased desire to save leads to a fall in the risk-free real interest rate. It also causes households, when deciding the composition of their savings portfolio, to require greater compensation to bear risk (that is, higher risk premiums), thus leaving the risky return on capital little changed.² The magnitude of the decline in the risk-free real interest rate and the offsetting rise in risk premiums depend on the source of income polarization, with the effects being especially large when greater inequality is caused by increased dispersion in returns on risky assets rather than by greater dispersion in labor income. Additionally, we illustrate how the implications for output and the capital stock also hinge on the nature of household income risk. Overall, the results underscore that the macroeconomic effects of inequality not only depend on the extent of income and wealth heterogeneity but also on households' ability to manage income risk through their portfolio decisions.

Most quantitative explorations of the links between inequality and the macroeconomy have focused on heterogeneous labor earnings as the source of inequality. This focus has been facilitated by the relative availability of data on labor compensation, which show that labor income dispersion across workers is not only pervasive, even when measured within demographic and skill groups, but it has increased over time. However, there is growing recognition that heterogeneity in labor income alone is insufficient to account for the observed extent of inequality and its evolution.³ In addition, the increased availability of panel data on household asset returns point to significant and persistent heterogeneity in households' saving behavior and the returns generated by their assets.⁴

¹These statistics are from Wolff (2021). Estimates of the extent and evolution of income and wealth inequality can be sensitive to data sources and definitions; see Bricker et al. (2016). Unless indicated otherwise, the household-level and aggregate concepts of "wealth" used throughout our paper are on a net rather than gross basis.

²Our analysis focuses on idiosyncratic risks. Aggregate risks may operate through similar channels at the household level.

³See, for example, Gabaix et al. (2016) and De Nardi and Fella (2017).

⁴For evidence on heterogeneous saving rates and risk tolerance, see Dynan, Skinner, and Zeldes (2004), Saez and

Furthermore, understanding the implications of income polarization requires the consideration of how household manage risk through their saving portfolio decisions, an aspect that is often missing from models that abstract from heterogeneous returns.

Given these observations, we reexamine the relationship between inequality, income risk, and the macroeconomy in a life-cycle model that allows for polarization in both labor income and asset returns across households.⁵ Our heterogeneous agents, overlapping-generation model builds on the Bewley-Huggett-Aiyagari (BHA) framework. As is typical with this framework, we assume that households are subject to persistent idiosyncratic labor income shocks each period until they reach retirement. Thereafter, they rely on their savings and social security transfers to sustain consumption. We extend the BHA framework by positing that households can allocate their savings to two kinds of assets. A safe asset—modeled as a one-period bond—offers the same deterministic return to all households. A risky asset—modeled as physical capital—offers a stochastic return that, in contrast to most previous studies, varies both across households and over time. 6 Households face a cost of adjusting their holdings of the risky asset, making that asset less liquid than the risk-free asset and inducing most households to hold both types of assets in equilibrium. The production side of our model is neoclassical. Therefore, the effect on economic activity and the relationship between measured inequality and output ultimately depend on how income risk affects incentives to save and how much of that saving is apportioned to the risky capital asset. We examine the model's general equilibrium in which all aggregate prices and quantities are determined endogenously, including in asset markets.

To explore the longer-run macroeconomic implications of various potential drivers of inequality, we compare the stationary equilibrium under our baseline calibration with the stationary equilibria under four alternative parameterizations that lead to equal-size increases in wealth inequality, as measured by the Gini coefficient of the distribution of wealth. In our first and second alternative parameterizations, we boost inequality in labor income across households by raising the volatility and persistence, respectively, of innovations to labor income. In our third and fourth alternative parameterizations, we similarly increase inequality in asset income across households by raising the volatility and persistence, respectively, of innovations to households' returns on the risky asset. Each of these four alternative parameterizations creates extra dispersion in total household income and, ultimately, greater concentration of wealth among the richest households.

A key finding is that increased income polarization, as captured by our four alternative param-

Zucman (2016), Bach, Calvet, and Sodini (2018), Crawley and Kuchler (2020), and Sias, Starks, and Turtle (2020). For panel-data evidence of differences in asset returns across households, see Fagereng et al. (2020), Bach, Calvet, and Sodini (2020), and Gomez (2018).

⁵Other possible contributors to inequality not explored in our study include inheritances (for example, Wolff and Gittleman (2014)), heterogeneous preferences (for example, Krusell and Jr. (1998), Toda (2019), and Carroll et al. (2017)), heterogeneous household labor supplies (for example, Flavin and Yamashita (2002; 2011) and Yum (2018)), and changes in tax rates and transfers (for example, Kaymak and Poschke (2016) and Cao and Luo (2017)). Because our approach to modeling labor income and asset returns is somewhat reduced form, the effects of other contributing factors may be conflated to some extent with the channels that we model.

⁶One exception is Guvenen, Kambourov, et al. (2019), who introduce idiosyncratic returns to saving through a three-state entrepreneurial shock.

eterizations, is a plausible contributor to increased income and wealth inequality and the fall in safe interest rates observed in recent decades in the United States and other advanced economies. In particular, greater riskiness in labor income or asset income in our parameterizations amplifies precautionary motives, which, in turn, boosts the demand for the safe asset and lowers the risk-free real interest rate. It also leads to an increase in the risk premium that households require as a compensation for bearing risk. On net, the rise in the risk premium offsets, in large part, the decline in the risk-free real interest rate, leaving the expected real rate of return on capital little changed across alternative parameterizations. This finding is consistent with the relative stability of the realized rate of return on capital in recent decades, as documented by Gomme, Ravikumar, and Rupert (2011). Accordingly, increased inequality is a plausible contributor to the observed divergence between the risky and the risk-free real interest rates.⁷

Though the direction of these effects is the same across alternative parameterizations, their magnitudes differ considerably depending on the origins of a rise in inequality. We find the largest swings in the risk-free real interest rate and risk premium when greater inequality originates from changes in the risky asset return process. Intuitively, households' ability to mitigate income risk through savings in capital units is curtailed when the return on physical capital has greater perperiod volatility or persistence. Accordingly, when inequality originates from greater polarization in risky asset returns, households seek to channel a larger share of their savings toward the safe asset. Households' desire to rebalance their portfolios further lowers the risk-free real interest rate and raises the risk premium in equilibrium. We also find that changes in inequality that are caused by increases in the persistence of the labor income and risky return processes have smaller macroeconomic effects than changes caused by increases in the per-period volatility of innovations to these processes. Although both kinds of changes lead to greater income and wealth polarization in the long run, changes caused by increases in persistence make household income more predictable over short horizons; therefore, the precautionary motives that drive macroeconomic effects are not as pronounced as when greater income polarization originates from more volatile innovations.

In contrast to the effects on the risk-free real interest rate and risk premium, which go in the same directions across the four parameterizations, the signs of the effects on the capital stock and real output vary. When the labor income process is more volatile, households accumulate more savings in both assets and the higher level of physical capital leads to higher output. When the return on capital is more volatile, the model instead predicts declines in the aggregate stock of capital and real output because households seek to rebalance their portfolios away from the productive capital asset. These results caution that the relationship between inequality and overall economic performance is complex and ultimately depends not only on the extent of wealth inequality, but also on how changes in income processes affect the incentives to invest in assets linked to the productive capacity of the economy.

In addition to the alternative parameterizations described in the previous paragraph, we consider

⁷For alternative explanations, see Caballero, Farhi, and Gourinchas (2017) and Marx, Mojon, and Velde (2019).

⁸The direction of these effects echoes those found in the Aiyagari (1994) model, in which physical capital is the only asset available to hedge labor income risk.

a fifth parameterization in which we accentuate wealth inequality by raising the correlation between innovations to the labor income and risky return processes, holding constant the volatility and persistence of each process. Raising the correlation leads to a higher aggregate stock of risky capital than under the baseline calibration because it means that households are more likely to enjoy high transitory income—and, thus, have a high desire to save—precisely when the return on the risky asset is most enticing for them. Accordingly, and in contrast with the results for the other four alternative parameterizations, increased wealth inequality in this parameterization is associated with a rise in the risk-free real interest rate and a fall in the risk premium.

To our knowledge, our paper is the first to explore the longer-run macroeconomic implications of inequality with a focus on the source of income risk in a general-equilibrium, overlapping-generations model that combines idiosyncratic labor incomes, idiosyncratic asset returns, and a portfolio allocation decision. As our results illustrate, these features interact in ways that make the origins of the observed rise in inequality relevant for inference about the macroeconomy. Other papers have embedded heterogeneous asset returns in BHA-type quantitative models. In partial-equilibrium settings, Benhabib, Bisin, and Zhu (2011) and Benhabib, Bisin, and Luo (2019) consider households that differ permanently from one another in terms of labor income and asset returns, creating income and wealth dispersion but no household-level income risk. These authors find the inclusion of heterogeneous returns to be important for capturing the extent of wealth inequality, especially at the top of the distribution. In a setting with only a risky asset, Pugh (2018) argues that heterogeneous returns is a compelling explanation for the joint evolution of wealth inequality and social mobility in the past few decades. The partial-equilibrium nature of these studies bars analysis of the macroeconomic implications.⁹

Our paper also connects to heterogeneous-agent models in which households face uncertain income streams and choose to allocate risk to both risk-free and risky assets but, in contrast to our study, otherwise enjoy identical returns within asset classes. Kaplan, Moll, and Violante (2018) and Ahn et al. (2017) use such models to illustrate how heterogeneity matters to the transmission of monetary and productivity shocks, respectively. Closer to our goal of studying longer-run macroeconomic effects, Cao and Luo (2017) apply such a model to the study of changes in tax policy and financial deregulation whereas Auclert and Rognlie (2018) explore the implications for aggregate output of heterogeneity in marginal propensities to consume. Our paper further connects with a burgeoning literature using large overlapping-generations models to study how demographics incentivize capital accumulation, thus leading to secular declines in interest rates. ¹⁰ These models are characterized by heterogeneity in wealth across generations (and, in some cases, heterogeneity in the labor supply) but the income received for a given amount of labor or savings are typically identical across households. Finally, our paper connects to studies of how households' ability to

⁹Before these contributions, Quadrini (2000) and Cagetti and De Nardi (2006) had focused on the role of entrepreneurs, who account for a significant share of the richest households and tend to enjoy relatively high asset returns.

¹⁰Among the contributors are Krueger and Ludwig (2007), Carvalho, Ferrero, and Nechio (2016), Kara and von Thadden (2016), Geppert, Ludwig, and Abiry (2016), Gagnon, Johannsen, and López-Salido (2021), Eggertsson, Mehrotra, and Robbins (2019), Jones (2018), and Auclert, Malmberg, et al. (2021).

manage risk relates to wealth inequality and interest rates. Our environment differs from the one in Favilukis (2013) in which improved access to capital markets leads to an increase in the aggregate stock of capital and depresses the return on capital and interest rates. Kasa and Lei (2018) allow for a portfolio choice between safe and risky assets in the context of Knightian uncertainty; they find that the robust portfolio choice helps explains the seemingly rapid rise in wealth concentration.

Our paper is organized as follows. Section 2 reviews the evolution of income and wealth inequality, interest rates, and risk premiums in the past few decades. Section 3 introduces our model. Section 4 describes the model's calibration and empirical fit. Section 5 presents the macroeconomic effects of various channels leading to higher measured inequality and relates our findings to those in the literature. Section 6 concludes.

2 Salient trends in income and wealth

2.1 Inequality benchmarks

In this study, we use Gini coefficients as simple benchmarks for the extent and evolution of income and wealth inequality in the population. As depicted in Figure 1, the Gini coefficient for total household income—the sum of labor income, asset income, and net transfers—rose from 0.48 in the 1983 SCF (the first such survey) to 0.57 in the 2019 SCF (the latest such survey), respectively. The corresponding Gini coefficient for aggregate net worth rose from 0.80 to 0.87 over that period. Movements in these Gini coefficients reflect, in a large part, an increasingly large share of income and wealth going to the most affluent households, as suggested by the strong comovement with the top 10 percent shares depicted in the figure. Inequality measures for net worth are markedly higher than those for total income, highlighting the greater concentration of wealth (a stock variable) than of income (a flow variable). Financial wealth (not shown), is even more concentrated than overall wealth. Changes in income and wealth inequality in the decades prior to the 1980s are somewhat mixed, with income-based measures somewhat up and wealth-based measures about unchanged on net.¹¹

2.2 Labor income polarization

Researchers have traced the rise in inequality in part to changes in the dynamics of labor income, which accounts for nearly 60 percent of national income and the lion's share of most households' income before retirement. These changes include soaring incomes among entrepreneurs, CEOs, superstars in entertainment and sports industries, investment bankers, lawyers, and members of a few more professions.¹² These high-earning occupations aside, labor income polarization in the general population has also grown, a phenomenon that researchers have linked to factors such

¹¹The pre-1983 estimates reported in Figure 1 are based on surveys that do not benefit from the same consistency in coverage and methodology as the SCF. Information from income tax returns suggests that wealth concentration was declining, on net, in the decades prior to the 1980s whereas the share of national income earned by the highest-earning households was broadly stable; see Saez and Zucman (2016).

¹²For facts and discussions of top inequality, see Gordon and Dew-Becker (2007).

as skill-biased technological change, a slowdown in growth of the supply of college graduates, a weakening of labor unions, and an erosion in the real minimum wage.¹³ Research suggests that this polarization reflects a rise in both the permanent and transitory components of labor income, as initially argued by Gottschalk and Moffitt (1994).

Several researchers have estimated labor income processes to understand how labor income varies over the life cycle and across households. Because the estimation of life-cycle parameters—notably the effects of age and experience on labor income—requires fairly long panels, researchers often make the identifying assumption that these parameters are constant over time. ¹⁴ By contrast, researchers interested in the evolution of income polarization often focus on a small number of characteristics, such as the college wage premium. Given our interest in both life-cycle elements and the evolution of polarization, it would be ideal to have elements of both approaches. However, because of the lack of detailed time-varying estimates, we find it most practical to consider a rich labor income process with fixed parameters and then pursue a model sensitivity analysis around key parameter values.

2.3 Asset return polarization

Though asset income accounts for a smaller share of national income than labor income, it may be an important contributor to rising inequality given the unequal distribution of assets in the population. Households have access to a broad range of investment vehicles, each with its potential risks and rewards. Households with similar income profiles may choose different asset allocations, resulting in different returns on their savings. Scandinavian administrative data, which include information on wealth, show that asset returns often differ significantly—and persistently so—across households even within asset classes, contributing to wealth inequality over time. Our model calibration of the returns to savings emphasizes this heterogeneity.

Our analysis draws on the evidence presented in Fagereng et al. (2020), who document the dynamics of the returns to savings among Norwegian households from 2004 to 2015. Their data set has several benefits relative to the corresponding U.S. data sets. First, they use administrative data that cover the entire population. Second, Norwegian authorities collect information on both asset holdings and asset income, whereas U.S. administrative data primarily seek to measure asset income (including capital gains, when realized). Finally, the long time coverage of the sample makes it possible to estimate the persistence of individual returns to savings. One potential drawback is that the Norwegian data need not be representative of the distribution of asset holdings and asset returns in the United States.¹⁶

¹³For evidence of increased dispersion in labor earnings after controlling for demographic and ability characteristics, see Lemieux (2006) and Barth et al. (2016). See also Katz and Autor (1999), Goldin and Katz (2007), and Autor, Katz, and Kearney (2008) for facts, discussions, and references on the drivers of growing labor income polarization.

¹⁴For example, see Kaplan (2012), Altonji, Smith Jr., and Vidangos (2013), or Pora and Wilner (2020).

¹⁵For evidence of differences in portfolio composition and average returns across U.S. households by wealth levels, see Saez and Zucman (2016) and Xavier (2021).

¹⁶For example, the top 10 percent of households owns about half of the total net worth in Norway, compared with almost 80 percent in the United States. In addition, public pensions, government transfers, and other income or consumption support programs are more prevalent in Norway than in the United States. Overall, it seems most likely that using Norwegian data in our calibration will understate the importance of heterogeneous returns for wealth

Fagereng et al. (2020) measure persistent differences in households' average asset returns by extracting fixed effects in yearly individual returns on net worth. They find much dispersion in these fixed effects: The standard deviation on (pre-tax) returns on net worth is 6.0 percentage points and the corresponding difference between the 90th and the 10th percentiles is 7.7 percentage points. Because these large differences in returns applied to an extended period, an investment in 2004 with a return equal to the 90th percentile of fixed effects would be valued nearly twice as much by 2015 as an equal investment with a return equal to the 10th percentile of fixed effects. The dispersion of annual returns is appreciably greater than that of fixed effects because household returns are also affected by transitory influences. Fagereng et al. (2020) report that the standard deviation of (pre-tax, uniformly weighted) returns on net worth is 22.1 percentage points whereas the 90th-10th percentile difference is 16.4 percentage points.

There also appears to be large and persistent differences in households' returns to savings in the United States. Using data from the Panel Study of Income Dynamics (PSID), which periodically include information on wealth, Cao and Luo (2017) report that the standard deviation in returns to wealth across households is 11.3 percent when they exclude capital gains and 27.1 percent when they include capital gains. These authors also report a coefficient of correlation of about 0.2 in five-year returns between the 1984–1989 and 1994–1999 periods whether they include capital gains or not. Saez and Zucman (2016) and Xavier (2021) also uncover evidence of economically significant differences in asset income by asset class in administrative and SPF data, respectively.

While it is clear that asset returns are polarized across households, and persistently so, the data are currently lacking on whether the degree of polarization has increased in past decades. Accordingly, and in contrast to the rise in labor income polarization, we see our model simulations of increased polarization in asset returns as exploratory of the mechanism.

2.4 Correlation between labor income and returns to savings

A further potential driver of inequality is the correlation between labor income and the returns to savings. To the extent that high-income earners also tend to enjoy relatively high returns on their savings, they will have an easier time amassing wealth than members of the general population. Saez and Zucman (2016) and Xavier (2021) offer evidence of a modestly positive correlation in the cross section of households. As with the evolution of polarization in asset returns, evidence is lacking on whether the cross-sectional correlation might have changed in a manner that contributed to the rise in inequality in past decades. Given our interest in income risk and life-cycle decisions, our study focuses on the comovement between households' labor income and asset returns over time rather than on permanent differences across households, both of which could contribute to the observed correlation. Using PSID data and various definitions of labor earnings and risky asset returns, we find that the correlation between deviations from these variables' expected levels are near zero after controlling for socio-economic and cyclical factors. Appendix A provides the detail of our empirical analysis, along with a discussion. Accordingly, in our baseline model calibration,

inequality in the United States.

we shall make the simplifying assumption that the underlying processes for labor earnings and asset returns are uncorrelated. In our subsequent model analysis, we explore the consequences for measured inequality and the macroeconomy of introducing some positive correlation.

2.5 Trends in returns on safe and risky assets

The top panel of Figure 2 shows time-series evidence of the (short-run) real risk-free real interest rate in the United States.¹⁷ All but one estimate are roughly flat from the 1960s through the 1980s, a period when measures of wealth inequality were little changed, on net.¹⁸ From the 1980s onward, estimates of the risk-free real interest rate are generally declining, on net, while measures of income and wealth inequality are steadily rising. Understanding the causes of this fall is of chief importance to central banks because, all else being equal, a low risk-free real interest rate is associated with a higher-than-otherwise probability that policy rates will be constrained by their effective lower bounds.

By contrast, the lower panel shows no apparent trend decline in estimates of the return on productive capital over the post-war period, as estimated by Gomme, Ravikumar, and Rupert (2011). If anything, the post-tax return on capital has moved somewhat higher, on net, since the 1980s, accentuating the gap with the risk-free real interest rate as measured wealth inequality rose. These observations are consistent with a modest widening of the risk premium over the period. Thus, contrary to the assumption of the baseline neoclassical growth model, the risk-free real interest rate and the marginal product of capital have not moved in lock steps.

3 Model

Our general equilibrium model is based on the BHA life-cycle framework, which we extend by including idiosyncratic heterogeneity in the rate of households' returns on their savings in physical capital and a number of other features described below to improve the model's plausibility and empirical fit. Households face an uninsurable risk of death each period that is consistent with life-cycle tables. There are no state-contingent contracts to insure against stochastic fluctuations in labor earnings and returns on capital. However, households can mitigate the effects of these fluctuations on their consumption stream by acquiring or selling capital units and risk-free bonds, subject to borrowing constraints. Risk-free bonds represent claims on the government or indebted households. There is no aggregate risk, and households have perfect foresight of aggregate objects.

We assume that time is discrete and that the length of a period is one year. The timing of events is as follows. At the beginning of each period, new households enter the economy, some

¹⁷The time-series models are from Johannsen and Mertens (2018), Holston, Laubach, and Williams (2017), Lubik and Matthes (2015), Del Negro et al. (2017), Hamilton et al. (2016), and Christensen and Rudebusch (2019).

¹⁸The exception is the model of Holston, Laubach, and Williams (2017), in which the equilibrium real interest rate is assumed to comove one-for-one with the trend in real GDP growth. This identifying assumption might contribute to high estimates of equilibrium real interest rates in the 1960s and 1970s, a period of high output and demographic growth, even though realized and expected risk-free real interest rates were low in those decades.

existing households die, and the assets of dead households are redistributed as accidental bequests to surviving households. The stochastic components of labor earnings and returns to physical capital are then realized. Surviving households finally decide how much to consume and save given the aggregate objects.

3.1 Problem of the household

The economy is thus populated by J overlapping generations of households aged 20 to J+20. The size of entering cohorts grows at a constant rate n. Households face a mortality risk that depends on their age, with $\Psi_{j,j'}$ denoting the probability that a household aged j survives to age j'. All households must retire at an exogenously set age j_r , after which they fund their consumption through their savings and social security. Households exogenously supply h_j units of labor, which is constant at \bar{h} over their productive years and drops to zero upon retirement. The problem of household i that is aged j in period t is to maximize

$$E\left[\sum_{\tau=0}^{J-j} \beta^{\tau} \Psi_{j,j+\tau} U\left(c_{i,t+\tau}\right)\right],\tag{1}$$

where β is a constant discount factor and $U(c_{i,t}) = (c_{i,t})^{1-\gamma}/(1-\gamma)$ is the per-period utility associated with household *i*'s consumption in period *t*, with γ being the intertemporal elasticity of substitution. The maximization is subject to a sequence of budget constraints,

$$c_{i,t+\tau} + a_{i,t+\tau+1} + P_{t+\tau}b_{i,t+\tau+1} + Adj(a_{i,t+\tau+1}, a_{i,t+\tau}) = W_{t+\tau}h_{j,t+\tau}\omega_{i,t+\tau} + a_{i,t+\tau}(1 + \widetilde{r}_{i,t+\tau}) + b_{i,t+\tau} - \Gamma_{t+\tau}\left(y_{i,t+\tau}^{l}, y_{i,t+\tau}^{s}\right) + beq_{t+\tau} + ss_{i,t}.$$

In this expression, $a_{i,t}$ and $b_{i,t}$ represent household i's holdings at the start of period t of risky physical assets and risk-free bonds, respectively. P_t is the price of the one-period bond, which pays a unit of consumption in period t+1. These expenses are funded through a number of income sources. First, the household receives labor income from supplying $h_{j,t}$ units of labor with household-specific productivity $\omega_{i,t}$ at the wage rate W_t per efficient unit. Households also receive income from their investments in physical capital, which has a household-specific rate of return net of depreciation of $\tilde{r}_{i,t}$, and in risk-free bonds. Households resources are complemented by accidental bequests, beq_t , and, after retirement, social security payments, $ss_{i,t}$. Households must pay taxes on their labor income and net income on their savings. In addition, households pay an adjustment cost when they expand or draw down their holdings of physical capital,

$$Adj(a_{i,t+\tau+1}, a_{i,t+\tau}) = \Theta_1 (a_{i,t+\tau+1} - (1 + \widetilde{r}_{i,t}) a_{i,t+\tau})^{\Theta_2}.$$

This cost formulation is similar to the adjustment cost on illiquid assets in Kaplan, Moll, and Violante (2018).

For convenience, we express the household i's labor income, $W_t h_{j,t} \omega_{i,t}$, and (net) asset income,

 $\widetilde{r}_{i,t}a_{i,t} + \left[\frac{1}{P_{t-1}} - 1\right]b_{i,t}$, as $y_{i,t}^l$, and $y_{i,t}^s$, respectively. The household pays taxes on labor and asset income according to the tax function $\Gamma\left(y_{i,t}^l, y_{i,t}^s\right)$ to be detailed later. Finally, household maximization is subject to a positivity constraint on risky asset holdings (that is, $a_{i,t} \geq 0$) and a requirement that households enter retirement with no debt (that is, $b_{i,t} \geq 0$ if $j \geq j^{ret}$).

3.2 Processes for labor earning and physical capital returns

Our choice of a labor income process is based on the parsimonious approach of Kaplan (2012), which is fairly standard in the literature and can be approximated with reasonable accuracy using a discrete state space with a limited number of states. He assumes that the process for the log of a worker's labor productivity depends on four additive terms,

$$\log\left(\widetilde{\omega}_{i,t}\right) = \varepsilon_i + \alpha_i + \nu_{i,t} + \theta_{i,t}. \tag{2}$$

The first term is an age-specific productivity component (with j being short-hand for the age of individual i in period t) that evolves deterministically over a household's lifetime. The second term is a permanent component to labor productivity that is realized when a worker enters the economy. The last two terms are persistent and purely transitory shocks. The persistent component is assumed to evolve according to a first-order autoregressive process,

$$\nu_{i,t} = \rho \nu_{i,t-1} + \epsilon_{\nu,i,t},$$

where $\epsilon_{\nu,i,t} \sim NID\left(0,\sigma_{\nu}^{2}\right)$ is an innovation that is normally and independently distributed. The transitory shock is similarly i.i.d., with $\theta_{i,t} \sim NID\left(0,\sigma_{\theta}^{2}\right)$.

We model differences in rates of return on capital in a similar manner. We assume persistent differences in households' ability to grow their capital holdings over time, with

$$\widetilde{r}_{i,t} = R_t - \delta - \delta_{i,t}.$$

In this expression, R_t is the economy-wide marginal product of capital, δ is an economy-wide time-invariant depreciation rate, and $\delta_{i,t}$ is an idiosyncratic term that creates persistent differences in rates of return across households. Similar to the persistent labor income shock, we assume that the $\delta_{i,t}$ follows a mean-zero autoregressive process,

$$\delta_{i,t} = \rho_{\delta} \delta_{i,t-1} + \epsilon_{\delta,i,t},$$

where $\epsilon_{\delta,i,t} \sim NID\left(0,\sigma_{\delta}^2\right)$. Although we capture differences in rates of return through heterogeneity in the effective depreciation rates, we think of the above specification as standing-in for a variety of factors that might create such differences.

Our specification of the risky return as a persistent, but ultimately transitory, process differs from the identifying assumption in Fagereng et al. (2020) that the individual returns contain a

fixed-effect component. In practice, distinguishing between permanent and highly persistent (but ultimately transitory) differences in returns across households is challenging because the panels of individual returns are short. In the context of our study, the autoregressive (AR) assumption helps us illustrate how the volatility and persistence of returns influence the saving, consumption, and portfolio decisions of households in the model. In addition, we see persistent, but ultimately transitory, returns as qualitatively consistent with the observations of Pugh (2018) that there is significant churning at the top of the wealth distribution, with around one-third of the wealthiest 1 percent of households exiting this group every other year and being unlikely to return. ¹⁹

3.3 Representative firm

We assume that a representative firm operates a constant return to scale production function of the Cobb-Douglas form,

$$Y_t = AK_t^{\zeta} H_t^{1-\zeta}. \tag{3}$$

The firm rents labor and capital inputs from competitive markets, taking real wages and the real rental rate of capital as given. At the optimum, the following first-order conditions must be satisfied:

$$R_t = \zeta \left(\frac{H_t}{K_t}\right)^{1-\zeta} \tag{4}$$

and

$$W_t = (1 - \zeta) \left(\frac{H_t}{K_t}\right)^{-\zeta}.$$
 (5)

3.4 Government

Government spending, G_t , is assumed to be done in an unproductive sector and funded through taxes and debt issuance. The government's budget constraint is

$$G_t + B_t^g = P_t B_{t+1}^g + \int \Gamma_{t+\tau} \left(y_{i,t}^l, y_{i,t}^s \right) di.$$

The variable B_t^g represents the stock of one-period, risk-free public debt issued in period t-1 that matures in period t. We allow for past government spending to have resulted in a net supply of government debt. We normalize the value of public debt such that each unit sold in period t at a

¹⁹Of course, it is conceivable that a portion of differences in returns across households is permanent because of, say, differences across investors in their sophistication and risk tolerance, the existence of fixed costs of accessing and trading assets with relatively high returns, or differences in the economic fate of regions over long periods. The inclusion of permanent differences in asset returns in our model, as do Benhabib, Bisin, and Luo (2019) in a partial-equilibrium model with no income risk, would increase the importance of idiosyncratic returns for income and wealth inequality. However, it would also eliminate income risk as an explanatory factor for macroeconomic trends. Similarly, the inclusion of purely transitory shocks to asset returns might also boost income and wealth polarization.

price P_t pays one unit of consumption in period t+1. The tax function is

$$\Gamma_{t}\left(y_{i,t}^{l}, y_{i,t}^{s}\right) = \Upsilon_{1}\left(y_{i,t}^{l} + y_{i,t}^{s} - \left(\left(y_{i,t}^{l} + y_{i,t}^{s}\right)^{-\Upsilon_{2}} + \Upsilon_{3}\right)\right)^{-\frac{1}{\Upsilon_{2}}}.$$

The right-hand-side term is as in Gouveia and Strauss (1994) and captures tax policy in a simple but flexible manner. The parameters Υ_1 , Υ_2 , and Υ_3 allow taxes to be progressive, flat, or regressive and to feature a standard tax deduction. The specification embeds the assumption that, at the margin, labor income and asset income are taxed at the same rates.

The government also operates a pay-as-you-go social security system, taxing current workers and rebating the proceeds to current retirees. The associated budget constraint is

$$SS_t = \int \tau_{ss,t} y_{i,t}^l di,$$

where SS_t represents total social security disbursements to retirees (detailed in Section 4.2) and $\tau_{ss,t}$ is a flat tax rate on labor income. The inclusion of a social security regime improves the model's fit of consumption profiles during retirement. Absent such a regime, workers would face extra incentives to accumulate wealth to support consumption in retirement. Moreover, without social security, the consumption paths of retirees might eventually fall to unrealistically low levels because the rise in mortality risk boosts the rate at which they discount future consumption.²⁰

As noted in the description of the household problem, our model includes both government debt and private debt in the form of household borrowing and lending from each other. We assume that both types of debt are free of default risk and treated as perfect substitutes.²¹ By construction, the net supply of private debt is zero because each unit of private debt issued by a household is held by another household. Therefore, the sum of household holdings (both positive and negative) of risk-free assets must match the outstanding government debt,

$$B_t^g = \int b_{i,t} di.$$

4 Model calibration and fit

4.1 Equilibrium concept

Given our interest in the long-run relation between inequality, income risk, and the macroeconomy, our analysis considers the stationary competitive equilibrium of the model, which we formally define in Appendix B. In a nutshell, we assume that the economy has converged to an equilibrium in which the proportion of households across all possible states—age, safe and risky asset holdings, labor income shocks, and returns-to-savings shocks—is constant over time. Firms and households have

²⁰The inclusion of a social security regime can also affect the normative aspects of the model. See Peterman (2013) for an illustration of how excluding social security affects optimal tax policy.

²¹We assume that the risk-free bond holdings of deceased households are distributed to survivors whether these holdings are positive or negative. Accordingly, there is no default on the negative balances of deceased households.

rational expectations and behave optimally, and the government's revenues match expenditures on social security, public consumption, and debt service. All markets clear, including the markets for factors of production that underpin the level of output and interest rates.

4.2 Calibration

Table 1 summarizes the parameter values in our baseline calibration. Roughly speaking, the parameters can be split into three broad groups: the processes for labor income and asset returns, which are at the core of our analysis; the parameters belonging to household preferences, demographic features, production, and taxation whose values we take from other studies; and the remaining parameters whose values we select to match specific moments of the data.

Our calibration of the labor income process is based on a discretization of the process estimated by Kaplan (2012), shown in Equation 2. This process includes an age-specific component, which is deterministic and common across households, and three stochastic components that vary across households. We assume that the permanent ability component (α_i), which is determined at the start of a household's life, can take two values (α_L or α_H). We also assume that the i.i.d. component ($\theta_{i,t}$) can take two values each period (θ_L or θ_H). Finally, following Kopecky and Suen (2010), we use the Rouwenhorst method to approximate the autoregressive persistent component using five discrete states. We further assume that the persistent component in the first period of adulthood, $\nu_{20,t}$, is always zero—the middle state. Consistent with the data, this latter assumption implies that the variance of labor earnings is mildly increasing with age.

Our calibration of the process guiding individual returns on the risky asset is also based on discretization of an autoregressive process with normal innovations. We set the persistence of annual returns, ρ_{δ} , to 0.9. We then set the volatility of innovations to idiosyncratic depreciation rates, σ_{δ}^2 , such that the unconditional (long-run) standard deviation of the autoregressive process is 3.59 percent. This value corresponds to the sample-wide standard deviation of estimated fixed effects in household-level returns to financial wealth reported in Fagereng et al. (2018).²² We see this calibration as capturing the important feature that individual returns can differ persistently across households. That said, we also see the calibration as illustrative rather than as a precise depiction of household-level differences in asset returns, for a couple of reasons. First, the mapping between our two-asset framework and the rich variety of assets available to households is stylized. Second, the imputation of marginal rates of return in household level-data is challenging in practice.²³ On the whole, we think of our calibration as more likely than not to underestimate the dispersion in actual asset returns across households. In part, the long-run dispersion in the data includes not only the dispersion in fixed-effect components, but also the effects of transitory shocks. And though

²²We use a statistic from the working-paper version of Fagereng et al. (2020) as calibration target because the definition of financial wealth in the published version of the paper changed in a manner that maps less directly into the two-asset structure of our model.

²³Fagereng et al. (2020) focus on realized returns on positive asset positions. They observe assets for several asset classes, including elements such as checking accounts for which nearly all households have positive balances. Accordingly, they are able to derive overall rates of returns for households that have no or negative wealth. Whether these rates of return are representative of households' investment opportunities is unclear.

our process is persistent, it does not yield permanent differences in returns.²⁴ In addition, the targeted returns on financial assets exclude entrepreneur income, which is a significant source of capital income dispersion.

Under our baseline assumptions, the autoregressive processes for labor income and for the rate of return on the risky asset are independent of each other. However, one might expect the two processes to be positively correlated. As we discussed in Section 2.4, we find little, if any, evidence of a positive correlation between the two processes in PSID data. However, the PSID sample has a poor coverage of high-earnings households for which a positive correlation arguably might be most likely to arise. Accordingly, in one of the alternative parameterizations considered in Section 5, we instead assume that the two processes are correlated. To do so, we use the method of Farmer and Toda (2017), an efficient procedure that allows for correlation while preserving the baseline mean and variance of the marginal processes. Labor income and asset returns could also be permanently higher for relatively rich households than for relatively poor ones, while not displaying correlation over time. For example, high-ability households might enjoy relatively high asset returns, on average, than low-ability households. While this situation would meaningfully contribute to inequality, it would not be necessarily associated with greater income risk, which is the object of our study. For simplicity, we also assume away permanent differences in mean returns across households.

Our functional choices and calibration of household preferences, the production function, and demographic variables follow Peterman (2013). We set the mandatory retirement age (j^{ret}) to 66 years, the maximum life span (J) to 100 years, and the labor supply of workers (\bar{h}) 1/3. The survival probabilities are consistent with the mortality rates for men reported by Bell and Miller (2005). For taxation, we take the key parameters guiding the general level and progressiveness of income taxes $(\Upsilon_1$ and $\Upsilon_2)$ from Gouveia and Strauss (1994). We also assume that, for each permanent ability type, social security payments equal 40 percent of labor income, averaged over the productive years of all workers of each type. This replacement rate is consistent with the evidence presented in Huggett and Parra (2010).

Finally, given the previous parameter assumptions, we calibrate the remaining parameter values to approximate various moments of the data. We set the discount factor (β) so that the aggregate capital-output ratio is 2.7.²⁶ We choose the remaining tax function parameters (Υ_3 and τ_{ss}) such that government expenditures and government social security payments equal receipts from general

²⁴An alternative calibration approach, pursued by Castañeda, Díaz-Giménez, and Ríos-Rull (2003), is to posit flexible processes and let the model select those processes so as to best match the empirical distributions of income and wealth. These authors show that, in a BHA model with inter-generational linkages, a labor income process that features considerable dispersion, skewness, and persistence can replicate the U.S. income and wealth distributions "almost exactly." However, this apparent success comes with an important drawback: The contribution to inequality of any other source of heterogeneity can be mistakenly attributed to labor income heterogeneity. Benhabib, Bisin, and Luo (2019) follow a similar approach with both heterogeneous labor income and heterogeneous asset returns. We follow the more typical approach in the literature by parameterizing relatively simple processes to the observed dynamics of the driving variables.

²⁵See Straub (2019) and Platzery and Peruffo (2021) for explorations of the role of increased permanent differences in labor income on the general level of interest rates.

²⁶This value corresponds to the ratio of fixed assets and consumer durable goods, minus government fixed assets, to GDP reported by Conesa, Kitao, and Krueger (2009).

income taxes and social security taxes, respectively. Finally, the risky asset adjustment cost parameters (Θ_1 and Θ_2) and borrowing limit (\underline{a}) are chosen to approximate three moments of the wealth distribution from the 2013 SCF. The first moment is the economy-wide share of total wealth that is invested in the risk-free asset.²⁷ The second moment is the gap between the respective shares of wealth held in the safe asset by the low- and high-ability types, which we assumed to be comparable to the difference in the shares for people with and without a high school diploma. The third moment is the fraction of households under the age of 65 who have negative net worth.

4.3 Fit of targeted and non-targeted moments

Our calibration matches the targeted capital-output ratio and ensures that government spending and social security disbursements are matched by corresponding tax revenues. Our model also approximates the three targeted SCF moments. On average, households in our model hold 19.6 percent of their wealth in the safe asset, compared with 19.4 percent in the SCF. The share of wealth held in the safe asset by the high-ability type in the model is 8.0 percentage points higher than for the low-ability type, compared with a difference of 11.4 percentage points between those with and without a high school diploma. The fraction of households aged 65 years or less with negative wealth, at 11.6 percent, is also near the fraction in the SCF, at 12 percent.

We further validate the model by checking its fit of moments that we do not directly target. Figure 3 demonstrates that the model does a reasonable job of matching the average share of wealth in the risk-free asset over the life of a household, with negative values early in adult life originating from young households seeking to smooth life-time consumption. The model produces a risk-free real interest rate of 1.9 percent, which is within the range of empirical estimates. Moreover, the model produces a difference in returns of 3.0 percentage points between the risky asset and the safe bond, in line with the empirical estimates in Fagereng et al. (2020).²⁸

Table 2 details the model's fit of the income and wealth distributions. The model closely matches the distribution of income in the population, with the exception of the tails of distribution. In particular, the share of income going to the richest 5 percent of households in the model—and, especially, the richest 1 percent—understates the concentration of income at the very top of the distribution in the SCF. As a counterpart, the model overstates the share of income going to the poorest households. The associated Gini coefficient for income, at 38 percent, is 10 to 20 percentage points below the measure in the SCF, depending on the reference vintage. Unsurprisingly, the model matches the distribution of wealth less well than it matches the distribution of income, although,

²⁷We define risky assets as directly held stocks, non-financial assets (measured as the gross value of residential and nonresidential property, privately owned businesses, and other miscellaneous), along with one third of the value of individual retirement accounts/Keoghs plans. We further assume that mortgage debt corresponds to a negative position in the safe asset category, consistent with loans to individuals routinely being packaged into portfolios to mitigate idiosyncratic risks.

²⁸The absence of an equity premium puzzle in our model reflects in part various barriers to mitigating risks: incomplete markets, portfolio adjustment costs, and borrowing constraints. In addition, the consideration of depreciation shocks has been shown to help produce realistic risk premiums in related settings; see Hasanhodzic and Kotlikoff (2013).

when we exclude the top 5 percent of the distribution, the relative shares in the rest of the population are similar to those found in the data. The model also understates the fractions of household with zero or negative wealth. The model produces a Gini coefficient for wealth of 0.63, compared with 0.80 to 0.87 in the SCF. The introduction of heterogeneity in risky asset returns helps the model produce modestly more realistic levels of wealth inequality relative to a version of the model in which this heterogeneity is turned off (whose Gini coefficient is 0.60).

In large part, the model's understatement of the shares of aggregate income and wealth accruing to the richest households reflects the fact that the model was not designed to capture income dynamics specific to the richest few households. For simplicity, we have imposed the assumptions that idiosyncratic labor income and risky asset returns follow log-linear processes, whose calibration is largely based on time-series evidence for the general population. Empirical evidence such as Guvenen, Karahan, et al. (2015) and Fagereng et al. (2020) suggests extra skewness to the underlying processes that could exacerbate inequality if embedded in a nonlinear characterization of these processes.

5 Main results

5.1 Illustrative parameterizations

To explore the long-run relation between income inequality, income risk, and the macroeconomy, we compare the stationary equilibrium of our model under our baseline calibration to the stationary equilibria under five alternative parameterizations. Each alternative parameterization creates extra dispersion in total household income and, ultimately, greater wealth inequality in the population than under our baseline calibration. For comparability, we alter the calibration such that, under each alternative parameterizations, the Gini coefficient on wealth increases 1 basis point relative to its baseline value. In our first and second alternative parameterizations, we increase labor income polarization by boosting the volatility of innovations and persistence, respectively, of the autoregressive component of labor income. In our third and fourth alternative parameterizations, we similarly increase asset income polarization by boosting the volatility of innovations and the persistence, respectively, of the autoregressive process guiding households' returns on their holdings of the risky asset. In the final parameterization, we increase the correlation between the autoregressive processes for labor income and risky returns, but otherwise keep the variance and persistence of each process at their baseline values.

Table 3 lists the parameter values that generate those equal-size increases in wealth inequality. The table also reports the long-run variances of the two autoregressive processes, which are indicative of changes in the dispersion of labor income and returns on the risky asset in the population.²⁹ We

²⁹These long-run variances do not map one-for-one to the dispersion of income in the population for a few reasons, including that households in the model have finite lifespans and begin their life with the autoregressive components initialized at zero. This initialization leads to growing dispersion across households as a function of age/experience, a feature also found in the data (for example, Altonji, Smith Jr., and Vidangos (2013)). In addition, we assume that households are subject to both fixed ability effects and purely transitory disturbances, adding extra cross-sectional

note that significant increases in the volatility of innovations are required to boost the Gini coefficient for wealth by a single basis point. For labor income, we increase the variance of innovations by about a quarter (from 0.017 to 0.021), whereas, for risky asset returns, we nearly double that variance (from 0.007 to 0.013).³⁰ When we increase wealth inequality through greater persistence, changes in parameter values appear comparatively modest, with the autoregressive coefficient rising from 0.958 to 0.972 for labor income and from 0.90 to 0.93 for risky asset returns. However, Table 3 shows that the effects on the long-run variances of the AR processes are economically significant and, in the case of labor income, larger than when the increase in wealth inequality originates from more volatile per-period innovations. Somewhat in contrast to the above cases, raising the correlation between labor income and risky asset returns by only a small amount (from 0.00 to 0.02) is sufficient to raise wealth inequality by 1 basis point. This observation illustrates the potential for a positive correlation to explain a portion of observed income and wealth inequality.³¹

Table 4 shows that the parameterizations lead to a rise in the share of aggregate wealth that accrues to the richest households, with the richest 10 percent capturing essentially all of the relative gains. Roughly speaking, these relative gains occur at the expense of the middle classes, whereas the poorest households, who generally lack assets, see almost no change in their shares of aggregate wealth. On the whole, the direction of these changes is consistent with the evidence presented in Figure 1, which illustrated that the rise in inequality observed over the past several decades has been driven by the richest households claiming an increasingly large share of both aggregate income and aggregate wealth. We also note that a given increase in wealth inequality that originates from greater polarization in labor income tends to be more consequential for inequality in total income and consumption than if it originated from greater polarization in risky asset returns.

5.2 Summary of macroeconomic effects

Table 5 documents the associated changes in interest rates, risk premiums, the stock of outstanding risk-free bonds, the aggregate capital stock, and real output relative to their baseline values. For the first four alternative parameterizations, we find that increases in the per-period volatility and the persistence of either the labor income or risky return processes are associated with a lower risk-free real interest rate and higher risk premiums. The direction of the effects on the risk-free real interest rate and risk premiums is the same and consistent with observed trends whereas the magnitude of these effects varies depending on the origins of greater income inequality. For a given-size increase in the Gini coefficient, we find the largest effects on the risk-free real interest rate and risk premium when the increase originates from either a more volatile or more persistent risky asset return process. We also find that increases in wealth inequality associated with greater persistence

income dispersion beyond the autoregressive processes.

³⁰Equivalently, we raise the standard deviation of per-period innovations by about 10 percent for labor income and about 25 percent for risky returns.

³¹In this parameterization, the correlation is calculated with respect to time-varying processes. Similarly, assuming a positive correlation with regards to permanent differences in labor income and asset returns in the cross section of households could raise the extent of inequality in the model closer to that in the data.

of either the labor income process or risky return process tend to have smaller macroeconomic effects than corresponding increases caused by more volatile innovations to these processes. Like the first four alternative parameterizations, an increase in the correlation between the labor income process and the risky return process causes more dispersion in income and wealth. However, unlike these parameterizations, it is associated with a modest rise in the risk-free real interest rate and the risk premium.

Both the direction and size of the effects of increased income inequality on the aggregate capital stock, the risky rate, real output, and the quantity of outstanding risk-free bonds differ across the alternative parameterizations, cautioning that the relationship between income risk, wealth inequality, and overall economic performance is complex. As we detail below, the net effects on these other macroeconomic variables depend on how greater income risk affects the demand for capital and incentives to borrow and save over the life cycle.

5.3 Demand and supply of assets

To understand how changes in income inequality relate to macroeconomic objects in the model, Figures 4 and 5 report demand and supply curves for the two assets in the model. For convenience, we normalized these curves to be in deviations from baseline steady-state values. The curves are constructed as follows.

Starting with the market for the risk-free bond, we solve the household's life-cycle problem on a grid of risk-free real interest rate values. In doing so, we assume that each such value prevails in the current and all future periods. We hold all other aggregate objects constant at their baseline steady-state values. We obtain a demand curve for the risk-free bond under the baseline and alternative calibrations (labeled "Savers (baseline)" and "Savers (alternative)," respectively) by summing all positive holdings of the safe bonds across households. We similarly obtain supply curves for risk-free bonds (labeled "Borrowers (baseline)" and "Borrowers (alternative)") by first summing, for each value of the risk-free real interest rate on the grid, over all negative holdings of the safe bonds across households and by then adding government borrowing to this sum. For simplicity, we assume that real government borrowing as a fraction of GDP is the same under each alternative parameterization as under our baseline calibration.³²

Continuing with the market for the risky productive asset, we derive a supply curve (labeled "Savers (baseline)" and "Savers (alternative)" in the right panels) in the manner described in the previous paragraph, that is, by summing the holdings of physical capital across households for each value of the risky rate on the grid, holding all other aggregate objects constant, including the risk-free real interest rate. We derive a demand curve for physical capital (labeled "Marginal

³²Our simulations thus do not allow for the possibility that fiscal authorities would respond to changes in the risk-free real interest rate by permanently altering their debt-to-GDP ratios. How secular declines in the risk-free real interest rate have affected sovereign indebtedness over the past few decades is unclear. For the advanced economies—and reading through cyclical variations—the average debt-to-GDP ratio mostly moved sideways from the mid-1980s to before the pandemic-driven recession (see Poghosyan (2018)). This observation suggests that fiscal authorities have not responded to secular declines in risk-free real interest rates by systematically expanding fiscal deficits or consolidating public finances.

product of capital" in the right panels) from the first-order condition of the representative firm with respect to capital. The economy-wide risky rate corresponds to the rental rate of capital net of depreciation, $R_t - \delta$. Because we assume competitive factor markets, the rental rate equals the marginal product of capital, which is a decreasing function of the aggregate capital stock under our neoclassical production function (see Equation 4). Accordingly, the demand for physical capital is downward sloping and identical across the various parameterizations.

5.3.1 Greater labor income polarization

As suggested in the first row of Figure 4, an increase in the volatility of innovations to the autore-gressive labor income process heightens precautionary-saving motives, thus raising the demand for both the safe asset and the risky productive asset. In equilibrium, this overall increase in asset demand causes both the equilibrium risk-free real interest rate and the risky rate to fall. The higher demand for risky assets also causes the aggregate capital stock to expand. These consequences broadly accord with those found by Aiyagari (1994) in a model in which physical capital is the only asset available to direct increased precautionary savings resulting from an increase in labor income risk.³³ In our model, the fall in the risk-free real interest rate, at 26 basis points, is larger than the fall in the risky rate, at 9 basis points, resulting in a 17-basis-point increase in the risk premium. This increase reflects the greater compensation required by households to bear risk as a result of their more volatile income.

In contrast to the equilibrium aggregate stock of capital, which expands a little over 1 percent, the equilibrium quantity of risk-free bonds increases only a little relative to its baseline value. The relatively large price response and relatively muted quantity response in the risk-free bond market reflect in a large part the relatively steep supply of risk-free assets in the economy. In part, we find that the aggregate amount of household borrowing is fairly insensitive to changes in the risk-free rate. Moreover, government borrowing is assumed to be inelastic and accounts for the lion's share of the supply of risk-free assets.

When greater labor income inequality emanates from more persistent innovations, we again obtain greater dispersion in labor income across households. However, the precautionary motives are exacerbated to a lesser extent than under our first alternative parameterization because the extra persistence makes household labor income somewhat more predictable over the short to medium terms. Beyond the medium term, however, labor income is more uncertain than under the baseline calibration, reflecting the broader range of possible outcomes. A more uncertain labor income path in the distant future means, in turn, that the household is likely to make more frequent or larger adjustments (or both) to its stock of savings than otherwise. Because adjusting holdings of the risk-free bond is not, the risk-free asset is a relatively attractive investment to hedge the increased labor income risk beyond the medium

³³This channel is distinct from the one in Auclert and Rognlie (2017), who show that a rise in labor income heterogeneity stimulates aggregate capital formation because relatively rich households have lower marginal propensities to consume than relatively poor households.

term. Putting these various elements together, we find that the demand for the risk-free asset increases following an increase in the persistence of labor income, though not as much as in the first simulation, leading to a fall in the risk-free real interest rate. By contrast, the demand for the risky asset is essentially unchanged, as is the equilibrium risky rate, because the shift away from the risky capital asset to hedge long-term income risk offsets the increase demand coming from heightened precautionary motives.

5.3.2 Greater asset income polarization

Figure 5 shows the corresponding demand and supply of savings in response to increased polarization in asset income. All else being equal, when the per-period volatility of innovations to the risky return process increases, total household income is more uncertain, thus accentuating precautionary saving motives. In addition, all else equal, the greater riskiness of physical capital makes this asset a poorer instrument to hedge this extra income risk, causing households to seek to rebalance their portfolios by raising their demand for the risk-free asset while reducing their demand for the risky asset, leading to modest falls in the aggregate stock of capital and real output in equilibrium. The supply schedule of risk-free assets shifts down as households who would have sought to borrow under the baseline calibration are more reluctant to do so due to their more uncertain income and more limited ability to hedge risk. As a result of these movements in supply and demand, the risk-free real interest rate drops 75 basis points, the risky rate rises 10 basis points, and the risk premium jumps about 85 basis points in response to the rise in per-period volatility.

An increase in asset income polarization that originates from greater persistence in risky asset return also lowers the risk-free real interest rate much (almost 60 basis points) in our model, reflecting increased demand for the risk-free asset. The effect on the risky rate is comparatively small and negative on net. Intuitively, households with above-normal risky returns have more overall transitory income than normal, and thus a heightened desire to save. All else being equal, the increased persistence of the risky returns makes investing in physical capital more desirable than otherwise for these households. The persistence of returns also means that these households tend to have more wealth than other households on average. Households whose risky returns are lower than normal face the opposite incentives and tend to have less accumulated wealth than on average.³⁴ Thus, on net, the aggregate demand for capital rises modestly following an increase in persistence, leading to a small fall in the risky rate and increase in output in equilibrium.

As was the case for labor income, we find that the effects of increased polarization in asset incomes are a bit smaller when they originate from greater persistence than from larger volatility in per-period innovations to the autoregressive process. Again, the reason is that greater persistence makes asset income a bit more predictable than otherwise for a time, thus damping precautionary saving motives, even though asset incomes is riskier over extended periods.

³⁴In addition, households near or at the positivity constraint on capital holdings cannot decumulate the risky asset much, if at all, further limiting the offset.

5.3.3 Greater correlation between labor and asset income

Under a higher correlation between the autoregressive processes for labor income and the risky return, households tend to experience periods during which their labor income and asset returns are either both above or both below normal levels, thus exacerbating income and wealth inequality. Table 5 and Figure 6 show that a higher correlation is associated with a greater demand for the risky asset—and, thus, a higher capital stock in equilibrium than under the baseline calibration. A higher correlation also leads to a decrease in the demand for the risk-free asset, inducing a slight rise in the risk-free real interest rate and a narrowing of the risk premium. The direction of the effects on interest rates and the risk premium is opposite to the trend observed in the data and to the direction of these effects under the other four alternative parameterizations.

All else being equal, a higher correlation between labor income and the risky asset returns means that households want to channel a larger share of their savings toward the risky asset. Intuitively, under a higher correlation, households tend to have elevated transitory income—and, thus, a high desire to save—precisely when the return on the risky asset is advantageous, thus making the risky asset a relatively attractive investment vehicle. For households with adverse labor income and risky return realizations, the risky asset is a less attractive investment vehicle than otherwise. Because these households account for a smaller share of the supply of savings than households with positive shock realizations, in aggregate, the demand for the risky asset expands whereas the demand for safe asset declines.³⁵

5.4 Discussion

A motivating question for our paper is whether increased inequality is a plausible contributor to the observed fall in the risk-free real interest rate and widening gap between the risk-free real interest rate and the rate of return on capital in recent decades. Our short answer is a qualified "yes": In our first four alternative parameterizations, greater polarization in labor income or asset income generally leads to greater inequality in both income and wealth, a fall in the risk-free real interest rate, and a rise in the risk premium that leaves the return on physical capital little changed. That this constellation of effects is the same across these parameterizations and accords with the direction of observed changes suggests that, at least conceptually, these various trends could have been jointly influenced by a rise in income risk. In addition, as we discussed in Section 2, researchers have provided evidence that labor income has become more dispersed across households over time, even after controlling for socio-economic elements. The existence of a rise in asset income risk over time is more speculative because of the lack of household-level panel data with long time spans and information about asset positions and returns. That said, the rising prominence of entrepreneurs and other households reliant on asset income points to a possible role for polarized asset returns to

³⁵In addition, a higher correlation between labor income and the risky asset return makes overall income more dispersed over time, exacerbating the households' precautionary saving motives. While this effect stimulates overall savings, it is not strong enough in our model to fully offset the contraction in the demand for the risk-free asset due to portfolio switching toward the risky asset.

be a contributing factor to increased inequality. Our simulations suggest that even a small increase in asset income risk could affect the risk-free real interest rate and risk premiums notably, though other elements might be needed to fully account for the observed rise in inequality.

Of course, several factors other than increased income risk are possible contributors to greater inequality, the decline in the risk-free real interest rate, and increased risk premiums. For example, the most commonly cited explanations for the fall in the risk-free real interest rate are a step down in the pace of productivity growth and population aging. Auclert and Rognlie (2018) have also emphasized that, to the extent that relatively rich households have lower propensities to consume than relatively poor households, a redistribution of aggregate income toward the richest households results in downward pressure on interest rates. While these explanations help understand the decline in the risk-free real interest rate, they also imply that, absent other frictions and contrary to the data, the rate of return on capital should have fallen by an equal amount. Similarly, explanations for increased income and wealth inequality that emphasize greater permanent dispersion in income across households need not yield a divergence between risky and risk-free real interest rates. Alternative explanations for this divergence include economy-wide shortages of safe assets and shifts in risk attitudes that boost the demand for safe assets relative to capital.³⁶ The implications of these alternative explanations for inequality remain to be explored. To be sure, it could be that rising inequality, a falling risk-free real interest rate, and rising risk premiums are ultimately driven by separate rather than joint factors. To the extent that these phenomena are linked in some ways, however, we believe that explanations that emphasize shifts in risk, or in attitudes toward risk, are worth entertaining.

Another highlight of our analysis is that the relationship between inequality and the macroe-conomy depends on sources of inequality both in terms of size of the effects and, in cases, their direction. Under our four main alternative parameterizations, a given rise in inequality is most consequential when it originates from more polarized asset returns than more polarized labor income because relatively more of the precautionary asset demand is channeled toward the safe asset. It also tends to be less consequential when increased polarization in either labor income or asset returns is highly persistent because, all else being equal, the resulting increased predictability of household income in the near term alleviates precautionary motives. The possibility that the rise in inequality has been fueled by a rise in the correlation between labor income and asset returns would have opposite implications for the risk-free real interest rate and risk premiums under our first four alternative parameterizations. Whether an increase in this correlation is plausible is difficult to assess: To our knowledge, there is no reliable empirical description of the joint process for labor income and returns on savings to assess that correlation, let alone to measure how it might have changed over long periods. By contrast, as noted above, the possibility of greater polarization in either labor income or asset returns enjoys some empirical support.³⁷ These observations suggest

³⁶See Bernanke (2005) and Caballero and Farhi (2018).

³⁷The panel data set of administrative income and wealth data recently made available for Scandinavian countries offers hope that the joint process for labor income and asset returns could be estimated. However, these data sets are only available for the latest decade and half, a period that is arguably too short to assess how changes in this

that, at a minimum, a rise in correlation is unlikely to be a dominant influence on the observed rise in inequality.³⁸

The relationship between inequality measures and variables such as output, the capital stock, and the rate of return on capital is especially sensitive to the source of inequality.³⁹ Ultimately, the effects on these variables depends on households' incentives to accumulate the risky capital asset in the model. The demand for the risky capital asset, in turn, depends on how income risk affects households' precautionary motives—and, thus, their desire to accumulate savings—and how much of their savings they want to channel into the risky asset. Accordingly, the predicted effects on output, capital, and the risky return of a given change in inequality can be sensitive to assumptions about risk tolerance, the income and return processes, available asset classes, portfolio adjustment frictions, etc. The sensitivity of these effects might help understand why researchers have struggled to uncover a robust empirical link between inequality and real activity.⁴⁰

Finally, our closed economy abstracts from external influences on both the asset supply and asset demand schedules. In an open-economy context, these schedules could be more elastic, reflecting possibilities for transborder sales and purchases of assets, so that domestic developments in income risk and other factors might have lesser effects on interest rates and larger effects on the stock of outstanding assets than predicted by our model. That said, the rise in income and wealth inequality is a global phenomenon, as is the decline in the risk-free real interest rate, so that the model may nonetheless be informative about the macroeconomic effects of these global trends.⁴¹

6 Conclusion

We have argued in this paper that some of the factors behind the observed rise in income and wealth inequality are plausible contributors to the decline in the risk-free real interest rate as well as the widening gap between the risk-free real interest rate and the rate of return on capital over the past few decades. Quantitatively, the longer-run macroeconomic implications of changes in inequality

correlation might have affected the trend toward greater inequality.

³⁸In appendix C, we also illustrate that the degree of household income risk need not always be positively associated with income and wealth inequality by considering the effects of a reduction the volatility of in purely transient shocks to labor income. While such a reduction lowers income risk, it also makes households less reluctant to taking on debt, thus leading to increased dispersion in income and wealth in equilibrium. However, the macroeconomic implications of such a reduction are at odds with salient trends in recent decades, making it unlikely that this factor has been a dominant influence on inequality.

³⁹Auclert and Rognlie (2018) also emphasize the relationship between inequality and output is sensitive to the origins of inequality. While the conclusion is similar, their framework differs from ours along several dimensions. Notably, they consider infinitely lived households who can insure against transitory income shocks through a single asset that pays a predetermined rate of return, in contrast to our overlapping-generations setup with idiosyncratic income and asset return risk and portfolio allocation choice.

⁴⁰See Barro (2008) and Madsen, Islam, and Doucouliagos (2018). We also note that there is a strand of empirical literature exploring potential causal links between inequality and macroeconomic outcomes; for example, the possibility that declines in equilibrium interest rates might fuel inequality by raising asset valuations in a manner that disproportionately benefits wealthy households. This literature is focused on the persistent, but ultimately transitory, effects of monetary policy, as opposed to the long-run relationship in our study.

⁴¹On global trends in inequality, see Organisation for Economic Co-operation and Development (2014) and Hoffmann, Lee, and Lemieux (2020). On the global fall in interest rates, see King and Low (2014), Holston, Laubach, and Williams (2017), and Del Negro et al. (2019).

can differ depending on the source, with our model predicting that the risk-free real interest rate and risk premiums are especially sensitive to changes in the polarization of asset returns. Moreover, we find that the direction of the effect on output and the risky return are especially sensitive to the assumed source of inequality. Overall, our results point to the importance of incorporating realistic sources of income heterogeneity into heterogeneous-agent models, including on the asset return side, not only to better fit the income and wealth distributions, but also because their incorporation is material for the macroeconomic effects. In the same vein, we found that the inclusion of portfolio decisions, which capture how households manage income risk, is also material for macroeconomic predictions.

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A Correlation between labor income and risky asset return deviations in the PSID

Our baseline model calibration embeds the assumption of no correlation between labor income and risky asset returns. In this appendix, we show that the corresponding empirical correlation in the PSID is small, though data limitations and measurement error in the survey complicate precise inference.

The PSID includes information on a panel of households surveyed annually from 1968 through 1997, and every other year since then. The main portion of the survey collects a range of socioeconomic information about households, including labor and capital income. Household wealth and other balance sheet information, which is needed to calculate household-specific rates of returns on savings, has been collected since 1984 with the addition of the Wealth Supplement to the survey. This supplement was administered every five years through 1999, and then every other year subsequently.⁴² Together, the main questionnaire and the Wealth Supplement allow us to construct a measure of risky asset returns in every wealth supplement year and calculate its correlation with labor income.

Specifically, we define risky assets as the sum of net business equity, stocks, primary housing, and other real estate for which we observe the self-reported value in wealth supplement years. ⁴³ Because the PSID tracks households over time, we can define capital gains as the change in the value of asset holdings reported by a household between wealth supplements, net of any reported purchase or sale of assets. We define capital income as the sum of business and farm income, interest income, dividend income, and trust or royalty income in all years between wealth supplements. This definition allows us to compute annualized asset returns on risky assets between wealth supplement years as the sum of capital income and capital gains returns. As is standard in the literature, we focus on households that belong to the national sample drawn by the Survey Research Center (SRC sample). To limit the risk of measurement error driving the regression results, we drop observations (that is, combinations of a household and wealth supplement year) whose reported annualized risky asset returns since the previous wealth supplement year are either larger than 300 percent or less than negative 66.7 percent (equivalently, for which risky assets grew over threefold or fell to less than one-third of their original value at an annualized rate).

 $^{^{42}}$ We refer to years including a wealth supplement module as "wealth supplement years." These years are 1984, 1989, 1994, 1999, 2001, 2003, 2005, 2007, 2009, 2011, 2013, 2015, and 2017. Asset returns can be calculated for all these years except 1984.

⁴³This definition excludes self-reported cash and liquid assets, annuities and individual retirement accounts, motor vehicles, and other assets such as life insurance policies, valuable collections, trusts, etc.

To estimate the correlation between labor income and risky asset returns, we follow the methodology in the literature estimating labor income risk (for example, Kaplan (2012)). We first regress log labor income for the household head on a cubic age polynomial, education dummies, and time dummies to remove the common, aggregate components of labor income. The residuals provide a measure that combines permanent, persistent, and idiosyncratic deviations ($\alpha_i + \nu_{i,t} + \theta_{i,t}$) from mean labor income. We similarly remove the aggregate component of risky asset returns by regressing the risky asset returns of households on supplement year dummies and utilizing the residual as a measure of idiosyncratic deviations ($\delta_{i,t}$) to risky asset returns. We then measure the correlation between deviations in labor income with those in asset returns.

Figure A.1 reports the deviations in log labor income and risky asset returns for each combination of a household and supplement year in the sample, along with a linear regression line. These data suggest a positive, but small, correlation of 0.020 (with an associated p value of 0.01) between the two types of deviations. We cannot reject the hypothesis that the correlation coefficient is the same across supplement years. Thus, it seems unlikely that changes in this relationship are a significant driver of rising inequality in recent history. That said, the PSID's sample size is limited, particularly among high-wealth households, and the measurement of labor income and asset returns is inherently noisy. In addition, asset returns can be calculated in the PSID only since the late 1980's, by which time inequality measures were already rising steadily. To the extent that a small increase in the correlation has contributed to rising inequality, it is possible that this increase predates the start of our data.

B Definition of stationary competitive equilibrium

In this appendix, we define a stationary competitive equilibrium for the model. Household *i*'s state variables in period t are its age (j_i) , ability (α_i) , persistent labor income shock $(\nu_{i,t})$, idiosyncratic labor income shock $(\theta_{i,t})$, holdings of risky assets $(a_{i,t})$, holdings of safe assets $(b_{i,t})$, and return to savings in risky assets $(\tilde{r}_{i,t})$. We group these state variables into the vector x and let $\mu(x)$ be the measure of households in state x. For ease of notation, below we drop the dependence on x of the household's variables in the presentation of the optimization problem and in the aggregation across individuals. For a given set of exogenous demographic parameters, $(\{n, \Psi\})$, a government tax function $(\Gamma : \mathbb{R}^2_+ \to \mathbb{R}_+)$, social security benefits (ss), a production plan for the representative firm (N, K), and a household utility function $(U : \mathbb{R}_+ \to \mathbb{R}_+)$, a stationary competitive equilibrium consists of a household decision rule $\{c\}$ that is conditional on the state x, factor prices $(\{W, R\})$, accidental bequests (beq) received by surviving households, and the distribution of individuals across states $(\mu(x))$ such that the following holds:

1. Given prices, policies, accidental bequests, benefits, and that the labor income ω follows Equation 2, the household maximizes Equation 1 under rational expectations, subject to

$$c + a' + Pb' = W\omega h + (1 + \widetilde{r})a + b - \Gamma\left(y^l, y^s\right) + beq,$$

for $j < j^{ret}$, and

$$c + a' + Pb' = (1 + \widetilde{r})a + b - \Gamma\left(y^l, y^s\right) + beq + ss,$$

for $j \geq j^{ret}$. Additionally, for all households,

$$c \ge 0, \ 0 \le h \le 1, \ a \ge 0, \ a_1 = 0.$$

2. Prices R and W satisfy the firm's first-order conditions,

$$R = \zeta \left(\frac{H}{K}\right)^{1-\zeta} = \widetilde{r} - \delta$$

and

$$W = (1 - \zeta) \left(\frac{H}{N}\right)^{\zeta}.$$

3. The social security policies satisfy

$$\tau_{ss} = \frac{\int\limits_{i \mid j_i \geqslant j^{ret}} ss \ di}{\int\limits_{i \mid j_i < j^{ret}} \min\{hw\omega, \overline{y}\} \ di}.$$

4. Accidental bequests are given by

$$beq = \int (1 - \Psi)adi.$$

5. Government balances its budget

$$G + (1 - P) B^g + SS = \int \Gamma(y^l, y^s) di.$$

6. The markets for capital, labor, and goods clear, with

$$K = \int adi + beq,$$

$$H = \int h\omega di,$$

and

$$\int cdi + \int a'di + G = K^{\zeta}N^{1-\zeta} + (1-\delta)K.$$

7. The distribution of $\mu(x)$ is stationary, that is, the law of motion for the distribution of individuals over the state space satisfies $\mu(x) = Q_{\mu}\mu(x)$, where Q_{μ} is a one-period recursive operator on the distribution.

C Transient labor income volatility

In this appendix, we show that measured wealth inequality rises when all households face *less* labor income volatility of a purely transitory nature. This situation contrasts with all parameterizations described in the main text for which measured wealth inequality rises when household income risk—and thus measured *income* inequality—rises. A divergence between income inequality and wealth inequality is also at odd with the data, with both inequality measures having trended up in recent decades. That said, the experiment illustrates that greater measured inequality need not be associated with welfare losses or riskier income streams.

A reduction in transitory shock volatility makes households with few or no assets less fearful of negative income realizations that might drive their period consumption to very low levels. Because of a lessened precautionary motive, these relatively poor households are less averse to having low or negative asset holdings, as evidenced by the fall in the wealth share of the lowest two quintiles in Table 4. As a result, the wealth distribution widens. Figure A.2 further illustrates this phenomenon by reporting average risk-free bond holdings by age in the model. The reduction in transitory labor income volatility leads to increased borrowing by households in their 20s, followed by slower wealth accumulation by households in their 30s. Risk-free asset holdings catch up with their baseline level for households in their mid-50s, and then exceed that baseline level over the rest of the average household's life.

While this phenomenon accentuates measured wealth inequality, Table 4 shows that it also leads to significant declines in measured income and consumption inequality: The illustrative 1-basis-point increase in the Gini coefficient for wealth is associated with declines of 2.4 basis points and 0.7 basis point in the Gini coefficients for total income and consumption, respectively. This parameterization illustrates that increased income and wealth inequality need not be associated with increased consumption inequality. In that sense, to the extent that increased wealth inequality reflects an improvement in households' ability to smooth income fluctuations through adjustments in asset holdings, increased wealth inequality might be understood as leading to a general improvement in overall welfare.

Table 1: Baseline calibration of key model parameters

Parameter	Value	Description			
Household utility					
β	0.990	Household annual discount rate			
γ	2	Intertemporal elasticity of substitution			
$ar{h}$	0.333	Labor supply of workers			
Production function					
δ	0.083	Annual capital depreciation rate			
ξ	0.36	Share of capital in production			
Tax function					
Υ_1	0.258	Income tax function parameter			
Υ_2	0.768	Income tax function parameter			
Υ_3	0.098	Income tax function parameter			
$ au_{ss}$	TBA	Social security tax rate on labor income			
y^l	TBA	Maximum taxable labor income for social security			
Labor income					
α_L	-0.255	Permanent labor productivity: low value			
α_H	0.255	Permanent labor productivity: high value			
$\sigma_{ heta}^2$	0.081	Transitory shock: variance			
$ ho_{ u}$	0.958	Persistent shock: autoregressive coefficient			
$\sigma_{ u}^{2}$	0.017	Persistent shock: variance of innovations			
Risky asset return					
$ ho_{\delta}$	0.9	Risky asset returns: autoregressive coefficient			
σ_{δ}	0.0156	Risky asset returns: standard deviation of innovations			
$ ho_{ u\delta}$	0	Correlation between labor and risky return processes			
Portfolio					
<u>a</u>	-0.11	Borrowing limit			
Θ_1	0.5	Portfolio adjustment cost parameter			
Θ_2	1.8	Portfolio adjustment cost parameter			

Source: Authors' calibration.

Table 2: Statistics of the income and wealth distributions

		S	share by q	Share by quintile (percent)	rcent)				Gini	Households with
			1st		2nd	3rd	4t.h	5th	coefficient	negative wealth
	Top 1%	Next 4%	Next 5%	Next 4% Next 5% Next 10%						(percent)
$\underline{Income~distribution}$										
1. 1983 SCF	12.8	13.3	10.3	15.5	21.6	14.2	8.7	3.7	0.480	
2. 2019 SCF	19.0	16.9	10.6	14.6	18.3	11.0	6.9	2.7	0.571	
3. Baseline calibration	4.5	11.8	10.7	16.3	22.1	14.7	10.3	9.6	0.384	
4with constant risky returns	4.3	11.1	10.5	15.0	21.2	14.8	11.7	11.5	0.362	
Net wealth distribution										
1. 1983 SCF	33.8	22.3	12.1	13.1	12.6	5.2	1.2	-0.3	0.799	15.5
2. 2019 SCF	38.2	28.2	11.6	10.8	8.6	2.9	0.4	-0.8	0.869	19.6
3. Baseline calibration	7.3	19.1	16.1	21.9	23.8	9.5	2.4	0.0	0.630	9.3
4with constant risky returns	9.9	17.2	15.1	21.8	25.5	10.9	2.9	0.0	0.604	8.3

Source: Authors' calculations and Wolff (2021).

Notes: Under the baseline calibration with constant risky returns, the model is simulated with the decision rules for the baseline calibration but holding the realized risky return shock at zero.

Table 3: Alternative parameterizations

	Par	Parameter		Uncon	d. variand	Uncond. variances of AR processes	processes	Ch	Change in Gini coefficient (basis points)	ini coeffi	cient (bas	is points)
		Value	lue	Labor	Labor income	Risky	Risky return	Net	Labor Asset	Asset	Total	
Alt. parameterization	\mathbf{Symbol}	Bas.	Alt.	Bas.	Alt.	Bas.	Alt.	wealth	income	income	income	wealth income income income Consumption
Labor income AR process 1. Higher per-period variance	σ_{ν}^2	0.017	0.021	0.21	0.26	0.037	0.037	1.0	1.5	1.1	1.4	1.1
2. Higher persistence	ρ_{ν}	0.958	0.972	0.21	0.31	0.037	0.037	1.0	1.2	9.0	1.2	1.3
Risky return AR process 3. Higher per-period variance	92	0.007	0.013	0.21	0.21	0.037	0.068	1.0	0.0	6.4	1.0	0.3
4. Higher persistence	ρջ	0.90	0.93	0.21	0.21	0.037	0.052	1.0	0.0	3.5	0.7	0.3
Joint process 5. Higher correlation between labor income and risky return	$\rho_{\nu\delta}$	0.00	0.03	0.21	0.21	0.037	0.037	1.0	0.0	1.1	0.4	0.2

All symbols correspond to parameters of the model, with the exception of the correlation between the labor income and risky return Notes: All alternative parameterizations lead to a 1-basis-point rise in the Gini coefficient on wealth relative to the baseline simulation. processes $(\rho_{\nu\delta})$, which is measured using simulated data.

Table 4: Change in wealth distribution relative to baseline

	$^{ m Sh}$	are of agg	gregate we	Share of aggregate wealth by quintile (p.p. change)	intile ($\mathbf{p.p.}$ ch	nange)		Households with
Alt: narameterization			1st		2nd	2nd 3rd	4t.h	5th	negative wealth
- Landerson Company	Top 1%	Next 4%	Next 5%	Op 1% Next 4% Next 5% Next 10%		5			(p.p. change)
Labor income AR process 1. Higher per-period variance	0.28	0.70	0.2	-0.03	-0.56	-0.45	-0.14	-0.01	0.14
2. Higher persistence	0.42	0.82	0.32	0.05	-1.01	-0.51	-0.09	0.00	-0.12
Risky return AR process 3. Higher per-period variance	0.29	1.10	0.38	-0.12	-0.88	-0.63	-0.15	-0.01	0.02
4. Higher persistence	0.22	1.12	0.32	-0.1	-0.76	-0.58	-0.21	-0.02	0.20
Others 5. Higher correlation between labor income and risky return	0.2	0.64	0.29	0.03	-0.5	-0.46	-0.19	-0.02	0.35

Source: Authors' calculations.

Notes: All alternative parameterizations lead to a 1-basis-point rise in the Gini coefficient on wealth relative to baseline.

Table 5: Alternative parameterizations: macroeconomic effects

Alt. parameterization	Risk-free rate (b.p.)	Risky rate (b.p.)	Risk premium (b.p.)	Risk-free bonds (percent)	Capital (percent)	Output (percent)
Labor income AR process						
1. Higher per-period variance	-26.2	-9.0	17.1	0.13	1.09	0.40
2. Higher persistence	-9.4	-0.3	9.1	0.12	-0.10	-0.09
Risky return AR process 3. Higher per-period variance 4. Higher persistence	-74.8 -60.5	9.5 -2.6	84.2 58.0	-0.10 -0.02	-1.11 0.29	-0.40 0.10
Addendum 5. Higher correlation between labor income and risky return	6.7	-5.8	-12.5	0.10	0.68	0.24

Notes: The table reports deviations from values in the baseline calibration. All alternative parameterizations lead to a 1-basis-point rise in the Gini coefficient on wealth relative to baseline.

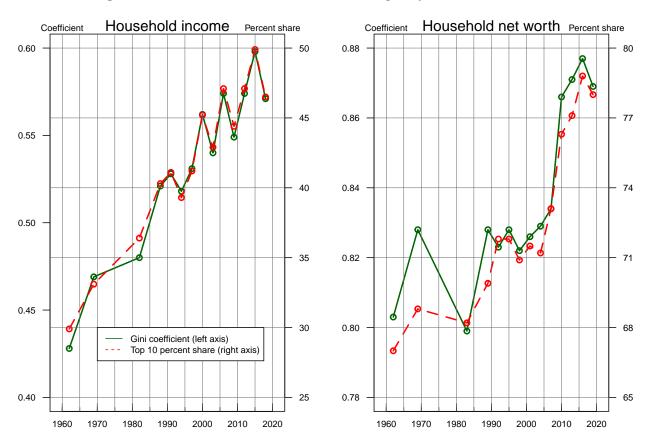
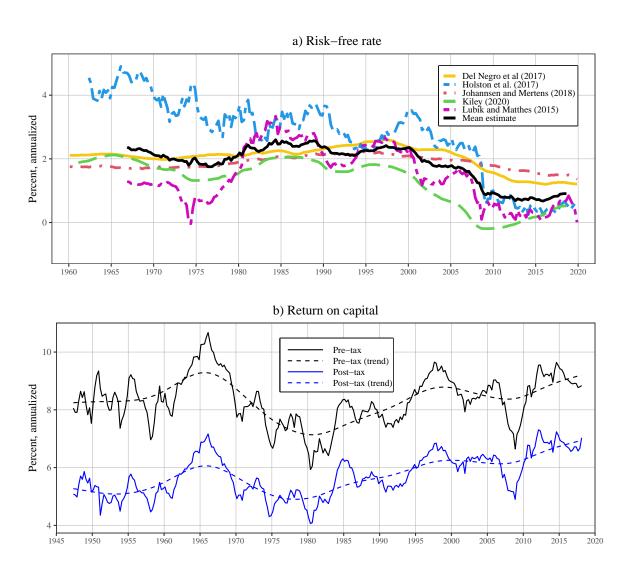


Figure 1: Indicators of income and wealth inequality in the United States

Sources: Wolff (2021) based on data from the Federal Reserve's 1962 Survey of Financial Characteristics of Consumers, the Census Bureau's 1969 Measurement of Economic and Social Performance project, and, starting in 1983, installments of the Federal Reserve's Survey of Consumer Finances. **Notes**: Households are ranked by their income in the top panel and by their net worth in the bottom panel for the calculation of the "top 10 percent share."

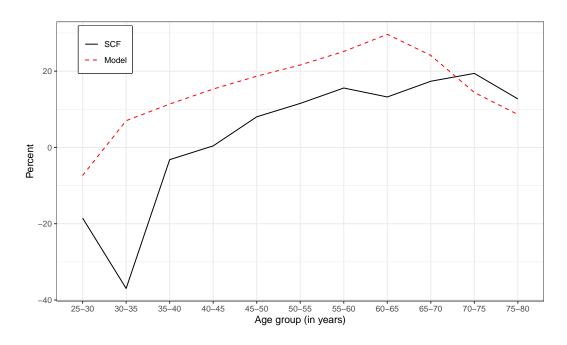
Figure 2: Estimates of the trend real risk-free real interest rate and real aggregate rate of return on capital



Sources: Updated estimates from Del Negro et al. (2017), Holston, Laubach, and Williams (2017), Johannsen and Mertens (2018), Kiley (2020), Lubik and Matthes (2015), and Gomme, Ravikumar, and Rupert (2011); authors' calculations.

Notes: The top panel shows time-series estimates of the trend real safe interest rate, along with the average of these estimates in each period. The lower panel reports quarterly estimates of the economy-wide rate of return on all types of capital, both on a pre-tax basis and an after-tax basis, as calculated by Gomme, Ravikumar, and Rupert (2011). The dashed lines show low-frequency trends using a Hodrick-Prescott filter with a Lagrange multiplier of 25,000.

Figure 3: Share of wealth in the risk-free assets



Source: Authors' calculations and 2013 Survey of Consumer Finances (SCF).

Higher per-period variance Risk-free bond Risky asset 100 Risk-free rate (basis-point deviation) 100 Risky rate (basis-point deviation) Savers (baseline) Marginal product of capita Savers (alternative) Borrowers (baseline) Savers (baseline) Borrowers (alternative) Savers (alternative) 20 20 0 0 -50 -50 -100 -100 2 2 -2 0 -2 0 4 4 Quantity (percent deviation) Quantity (percent deviation) **Higher persistence** Risk-free bond Risk-free bond 100 Risk-free rate (basis-point deviation) 100 Risky rate (basis-point deviation) 20 20 0 0 -50 -50 -100 -100 -2 0 2 4 -2 0 2 4 -4 -4 Quantity (percent deviation) Quantity (percent deviation)

Figure 4: Long-run effects on asset markets of greater dispersion in labor income

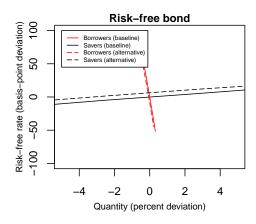
Notes: The panels show the long-run demand and supply of assets under the baseline calibration and two alternative parameterizations: a higher per-period variance (top row) and a higher per-sistence (bottom row) of the autoregressive labor income process. The alternative parameters are calibrated such that the Gini coefficient on wealth rises by 1 basis point relative to its baseline value. All curves are expressed in deviations from baseline equilibrium values. See Section 5.3 for details on the derivation of the curves.

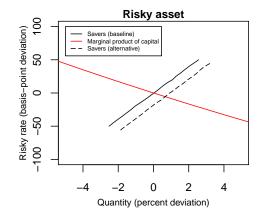
Higher per-period variance Risk-free bond Risky asset 100 Risk-free rate (basis-point deviation) 100 Risky rate (basis-point deviation) Savers (baseline) Marginal product of capita Savers (alternative) Borrowers (baseline) Savers (baseline) Borrowers (alternative) Savers (alternative) 20 20 0 0 -50 -20 -100 -100 2 2 -2 0 -2 0 4 4 Quantity (percent deviation) Quantity (percent deviation) **Higher persistence** Risk-free bond Risk-free bond 100 Risk-free rate (basis-point deviation) 100 Risky rate (basis-point deviation) 20 20 0 -50 -50 -100 -100 -2 0 2 4 -2 0 2 4 -4 -4 Quantity (percent deviation) Quantity (percent deviation)

Figure 5: Long-run effects on asset markets of greater dispersion in risky returns

Notes: The panels show the long-run demand and supply of assets under the baseline calibration and two alternative parameterizations: a higher per-period variance (top row) and a higher persistence (bottom row) of household returns on their risky asset holdings. The alternative parameters are calibrated such that the Gini coefficient on wealth rises by 1 basis point relative to its baseline value. All curves are expressed in deviations from baseline equilibrium values. See Section 5.3 for details on the derivation of the curves.

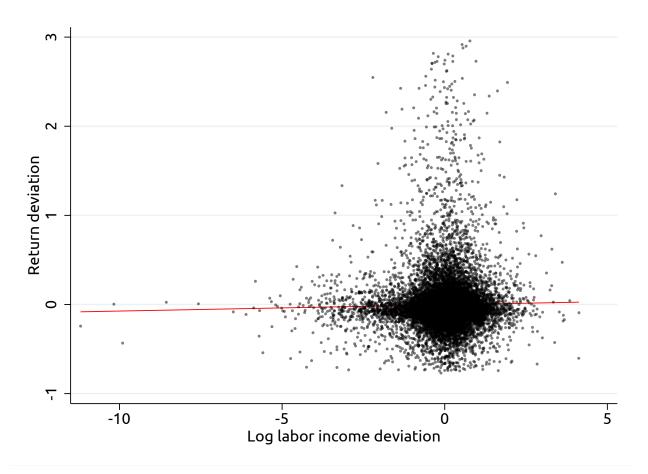
Figure 6: Long-run effects on asset markets of a higher correlation between labor income and risky returns





Notes: The panels show the long-run demand and supply of assets under the baseline calibration and the alternative parameterization with a higher correlation between labor income and households' returns on their risky asset holdings. The alternative parameters are such that the Gini coefficient on wealth rises by 1 basis point relative to its baseline value. All curves are expressed in deviations from baseline equilibrium values. See Section 5.3 for details on the derivation of the curves.

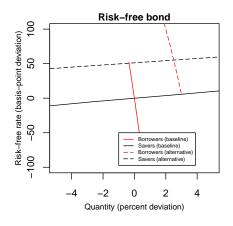
Figure A.1: Correlation between labor income and risky asset return deviations in the PSID

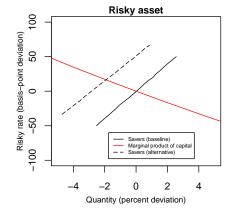


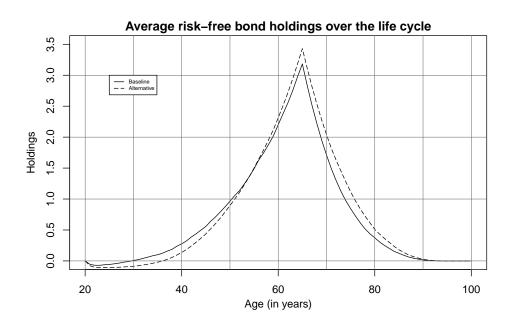
Sources: Authors' calculations using the SRC sample of the PSID.

Notes: The figure reports the deviation of annual labor income and annualized risky asset returns from their respective trends for all combinations of a household and wealth supplement year in the SRC sample. The line in red is a linear regression line. See Appendix A for methodological detail.

Figure A.2: Effect of lower transitory labor income volatility on average risk-free bond holdings over the life cycle







Notes: In the alternative parameterization, the volatility of purely transitory labor income shocks is reduced such that the Gini coefficient for wealth rises 1 basis point relative to its baseline value. The bottom panels show the long-run demand and supply of assets under the baseline calibration and the alternative parameterization. All curves are expressed in deviations from baseline equilibrium values. See Section 5.3 for details on the derivation of the curves.